

# K-Witt bordism in characteristic 2

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## Abstract

This note provides a computation of the bordism groups of  $K$ -Witt spaces for fields  $K$  with characteristic 2. We provide a complete computation for the unoriented bordism groups. For the oriented bordism groups, a nearly complete computation is provided as well a discussion of the difficulty of resolving a remaining ambiguity in dimensions equivalent to  $2 \pmod{4}$ . This corrects an error in the  $\text{char}(K) = 2$  case of the author's prior computation of the bordism groups of  $K$ -Witt spaces for an arbitrary field  $K$ .

In [1], an  $n$ -dimensional  $K$ -Witt space, for a field  $K$ , is defined<sup>1</sup> to be an oriented compact  $n$ -dimensional PL stratified pseudomanifold  $X$  satisfying the  $K$ -Witt condition that the lower-middle perversity intersection homology group  $I^{\bar{m}}H_k(L; K)$  is 0 for each link  $L^{2k}$  of each stratum of  $X$  of dimension  $n - 2k - 1$ ,  $k > 0$ . Following the definition of stratified pseudomanifold in [2],  $X$  does not possess codimension one strata. Orientability is determined by the orientability of the top (regular) strata. This definition generalizes Siegel's definition in [11] of  $\mathbb{Q}$ -Witt spaces (called there simply "Witt spaces"). The motivation for this definition is that such spaces possess intersection homology Poincaré duality  $I^{\bar{m}}H_i(X; K) \cong \text{Hom}(I^{\bar{m}}H_{n-i}(X; K), K)$ .

The author's paper [1] concerns  $K$ -Witt spaces and, in particular, a computation of the bordism theory  $\Omega_*^{K\text{-Witt}}$  of such spaces. However, there is an error in [1] in the computation of the coefficient groups  $\Omega_{4k+2}^{K\text{-Witt}}$  when  $\text{char}(K) = 2$ .

It is claimed in [1] that  $\Omega_{4k+2}^{K\text{-Witt}} = 0$ . When  $\text{char}(K) > 2$ , the null-bordism of a  $4k + 2$  dimensional  $K$ -Witt space  $X$  is established in [1] by following Siegel's computation [11] for  $\mathbb{Q}$ -Witt spaces by first performing a surgery to make the space irreducible and then performing

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<sup>1</sup>There is a minor error in [1] in that Witt spaces are stated to be irreducible, meaning that there is only a single top dimensional stratum. In general, this should not be part of the definition of a  $K$ -Witt space; cf. [11]. However, as every  $K$ -Witt space of dimension  $> 0$  is bordant to an irreducible  $K$ -Witt space (see [11, page 1099]), this error does not affect the bordism group computations of [1]. It is not true that every 0-dimensional  $K$ -Witt space is bordant to an irreducible  $K$ -Witt space, but in this dimension the computations all reduce to the manifold theory and the computations given for this dimension in [1] are also correct if one removes irreducibility from the definition.

a sequence of singular surgeries to obtain a space  $X'$  such that  $I^{\bar{m}}H_{2k+1}(X'; K) = 0$ . The  $K$ -Witt null-bordism of  $X$  is the union of the trace of the surgeries from  $X$  to  $X'$  with the closed cone  $\bar{c}X'$ . One performs the singular surgeries on elements  $[z] \in I^{\bar{m}}H_{2k+1}(X; K)$  such that  $[z] \cdot [z] = 0$ , where  $\cdot$  denotes the Goresky-MacPherson intersection product [2]. As the intersection product is skew symmetric on  $I^{\bar{m}}H_{2k+1}(X; K)$ , such a  $[z]$  always exists. The error in [1] stems from overlooking that this last fact is not necessarily true in characteristic 2, where skew symmetric forms and symmetric forms are the same thing and so skew-symmetry does not imply  $[z] \cdot [z] = 0$ .

**Corrected computations.** To begin to remedy the error of [1], we first observe that it remains true in characteristic 2 that the map<sup>2</sup>  $w : \Omega_{4k+2}^{\mathbb{Z}_2\text{-Witt}} \rightarrow W(\mathbb{Z}_2)$  is injective, where  $W(\mathbb{Z}_2)$  is the Witt group of  $\mathbb{Z}_2$  and  $w$  takes the bordism class  $[X]$  to the class of the intersection form on  $I^{\bar{m}}H_{2k+1}(X; \mathbb{Z}_2)$ . For  $k > 0$ , this fact can be proven as it is proven for  $w : \Omega_{4j}^{K\text{-Witt}} \rightarrow W(K)$ ,  $j > 0$ , in [1]: if one assumes that the intersection form on  $X$  represents 0 in  $W(\mathbb{Z}_2)$  then the intersection form is split, in the language of [7]; see [7, Corollary III.1.6]. And so  $I^{\bar{m}}H_{2k+1}(X; \mathbb{Z}_2)$  will possess an isotropic (self-annihilating) element by [7, Lemma I.6.3]. The surgery argument can then proceed<sup>3</sup>. As  $W(\mathbb{Z}_2) \cong \mathbb{Z}_2$  (see [7, Lemma IV.1.5]), it follows that  $\Omega_{4k+2}^{\mathbb{Z}_2\text{-Witt}}$  is either 0 or  $\mathbb{Z}_2$ .

This argument does not hold for  $4k+2 = 2$  as in this case the dimensions are not sufficient to guarantee that every middle-dimensional intersection homology class is representable by an irreducible element, which is necessary for the surgery argument; see [11, Lemma 2.2]. However, all 2-dimensional Witt spaces must have at worst isolated singularities, and so in particular such a space must have the form  $X \cong (\amalg S_i)/\sim$ , where the  $S_i$  are closed oriented surfaces and the relation  $\sim$  glues them together along various isolated points. But then  $X$  is bordant to  $\amalg S_i$ . This can be seen via a sequence of pinch bordisms as defined by Siegel [11, Section II] that pinch together the regular neighborhoods of sets of points of  $\amalg S_i$ . To see that the bordism is via a Witt space, it is only necessary to observe that the link of the interior cone point in each such pinch bordism will be a wedge of  $S^2$ s, and it is easy to compute that  $I^{\bar{m}}H_1(\vee_i S^2; K) = 0$  for any  $K$ . But now, since all closed oriented<sup>4</sup> surfaces bound,  $\Omega_2^{\mathbb{Z}_2\text{-Witt}} = 0$ . This special case was also over-looked in [1], though this argument holds for any field  $K$  and is consistent with the claim of [1] that  $\Omega_2^{K\text{-Witt}} = 0$  for all  $K$ .

Thus we have shown that  $w : \Omega_{4k+2}^{\mathbb{Z}_2\text{-Witt}} \rightarrow W(\mathbb{Z}_2) \cong \mathbb{Z}_2$  is an injection for  $k \geq 0$ , trivially

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<sup>2</sup>Recall from [1, Corollary 4.3] that the bordism groups depend only on the characteristic of the field, so for characteristic 2 it suffices to consider  $K = \mathbb{Z}_2$ .

<sup>3</sup>There is one other possible complication due to characteristic 2 that must be checked but that does not provide difficulty in the end: For characteristic not equal to 2, every split form is isomorphic to an orthogonal sum of hyperbolic planes [7, Lemma I.6.3], and this appears to be used in the proof of Theorem 4.4 of [11], which is heavily referenced in [1]. For characteristic 2, one can only conclude that a split form is isomorphic to one with matrix  $\begin{pmatrix} 0 & I \\ I & A \end{pmatrix}$  for some matrix  $A$ . However, a detailed reading of the proof of [11, Theorem 4.4, particularly page 1097] reveals that it is sufficient to have a basis  $\{\alpha, \beta, \gamma_1, \dots, \gamma_{2m}\}$  such that  $\alpha \cdot \alpha = \alpha \cdot \gamma_i = 0$  for all  $i$  and  $\alpha \cdot \beta = 1$ , and this is certainly provided by a form with the given matrix.

<sup>4</sup>Recall that  $\mathbb{Z}_2$ -Witt spaces are assumed to be  $\mathbb{Z}$ -oriented, though see below for more on orientation considerations

so for  $k = 0$ . Unfortunately, the question of surjectivity of  $w$  in dimensions  $4k + 2$  is more complicated and not yet fully resolved. We can, however, make the following observation: if  $X$  is a  $\mathbb{Z}_2$ -Witt space of dimension  $4k - 2$ , then<sup>5</sup>  $w([X \times \mathbb{C}P^2]) = w([X])$ . So if there is a non-trivial element of  $\Omega_{4k-2}^{\mathbb{Z}_2\text{-Witt}}$ , then there is a non-trivial element of  $\Omega_{4k+2}^{\mathbb{Z}_2\text{-Witt}}$ .

Putting this together with the computations from [1] of  $\Omega_*^{K\text{-Witt}}$  in dimension  $\not\equiv 4k + 2 \pmod{4}$  (which remain correct), we have the following theorem:

**Theorem 1.** *For a field  $K$  with  $\text{char}(K) = 2$ ,  $\Omega_*^{K\text{-Witt}} = \Omega_*^{\mathbb{Z}_2\text{-Witt}}$ , and for<sup>6</sup>  $k \geq 0$ ,*

1.  $\Omega_0^{K\text{-Witt}} \cong \mathbb{Z}$ ,
2. for  $k > 0$ ,  $\Omega_{4k}^{K\text{-Witt}} \cong \mathbb{Z}_2$ , generated by  $[\mathbb{C}P^{2k}]$ ,
3.  $\Omega_{4k+3}^{K\text{-Witt}} = \Omega_{4k+1}^{K\text{-Witt}} = 0$ ,
4. *Either*
  - (a)  $\Omega_{4k+2}^{K\text{-Witt}} = 0$  for all  $k$ , or
  - (b) *there exists some  $N > 0$  such that  $\Omega_{4k+2}^{K\text{-Witt}} = 0$  for all  $k < N$  and  $\Omega_{4k+2}^{K\text{-Witt}} \cong \mathbb{Z}_2$  for all  $k \geq N$ .*

We will provide below some further discussion of the difficulties of deciding which case of (4) holds after discussing unoriented bordism.

*Remark.* Independent of the existence or value of  $N$  in condition (4) of the theorem, the computations from [1, Section 4.5] of  $\Omega_*^{K\text{-Witt}}(\cdot)$  as a generalized homology theory on CW complexes continue to hold and to imply that for  $\text{char}(K) = 2$ ,

$$\Omega_n^{K\text{-Witt}}(X) = \Omega_n^{\mathbb{Z}_2\text{-Witt}}(X) \cong \bigoplus_{r+s=n} H_r(X; \Omega_s^{\mathbb{Z}_2\text{-Witt}}).$$

**Unoriented bordism.** Given the motivation to recognize spaces that possess a form of Poincaré duality, it seems reasonable to consider  $K$ -Witt spaces that are  $K$ -oriented. This has no effect when  $\text{char}(K) \neq 2$ , in which case  $K$ -orientability is equivalent to  $\mathbb{Z}$ -orientability as considered in [1]. But when  $\text{char}(K) = 2$ , all pseudomanifolds are  $\mathbb{Z}_2$ -orientable, which is equivalent to being  $K$  orientable, and the Poincaré duality isomorphism  $I^{\bar{m}}H_k(X; K) \cong \text{Hom}(I^{\bar{m}}H_{n-k}(X; K), K)$  holds for all such compact pseudomanifolds satisfying the  $K$ -Witt condition.

If we allow  $K$ -Witt spaces and  $K$ -Witt bordism using  $K$ -orientations, then for  $\text{char}(K) = 2$  we are essentially talking about unoriented bordism<sup>7</sup>, so to clarify the notation, let us denote the resulting bordism groups by  $\mathcal{N}_*^{K\text{-Witt}}$ . These groups can be computed as follows:

<sup>5</sup>Recall that the Künneth theorem holds within a single perversity when one term is a manifold, so we can compute the intersection forms of such product spaces in the usual way; see e.g. [6].

<sup>6</sup>Since these are geometric bordism groups, they vanish in negative degree.

<sup>7</sup>One could also define unoriented bordism groups of unoriented compact PL pseudomanifolds satisfying the  $K$ -Witt condition with  $\text{char}(K) \neq 2$ , but it is not clear how to study such groups by the present techniques, as there is no reason to expect that  $I^{\bar{m}}H_*(X; K)$  would satisfy Poincaré duality for such a space  $X$ .

**Theorem 2.** For a field  $K$  with  $\text{char}(K) = 2$  and for  $i \geq 0$ ,

$$\mathcal{N}_i^{K\text{-Witt}} \cong \begin{cases} \mathbb{Z}_2, & i \equiv 0 \pmod{2}, \\ 0, & i \equiv 1 \pmod{2}. \end{cases}$$

Since writing [1], the author has discovered that this theorem is also provided without detailed proof by Goresky in [4, page 498]. We provide here the details:

*Proof.* It continues to hold that the local Witt condition depends only on the characteristic of  $K$  for the reasons provided in [1], so we may assume  $K = \mathbb{Z}_2$ . To see that  $\mathcal{N}_n^{\mathbb{Z}_2\text{-Witt}} = 0$  for  $n$  odd, we simply note that  $X$  bounds the closed cone  $\bar{c}X$ , which is a  $\mathbb{Z}_2$ -Witt space. The map  $w : \mathcal{N}_{2k}^{\mathbb{Z}_2\text{-Witt}} \rightarrow W(\mathbb{Z}_2) \cong \mathbb{Z}_2$  is onto for each  $k > 0$ , as the intersection pairing on the  $\mathbb{Z}_2$ -coefficient middle-dimensional homology of the real projective space  $\mathbb{R}P^{2k}$  corresponds to the generator of  $W(\mathbb{Z}_2)$  represented by the matrix  $\langle 1 \rangle$ . Furthermore,  $w$  is injective for  $k > 1$  as in the preceding surgery argument, which does not rely on whether or not  $X$  is oriented, only on the existence of the intersection pairing over  $\mathbb{Z}_2$ . In dimension 0, we have unoriented manifold bordism of points, so  $\mathcal{N}_0^{\mathbb{Z}_2\text{-Witt}} \cong \mathbb{Z}_2$ . Finally, as in the argument above for  $\Omega_2^{\mathbb{Z}_2\text{-Witt}}$ , the group  $\mathcal{N}_2^{\mathbb{Z}_2\text{-Witt}}$  must be generated by closed surfaces (now not necessarily oriented), so  $\mathcal{N}_2^{\mathbb{Z}_2\text{-Witt}}$  is a quotient of the unoriented manifold bordism group  $\mathcal{N}_2 \cong \mathbb{Z}_2$ ; thus  $\mathcal{N}_2^{\mathbb{Z}_2\text{-Witt}}$  must be isomorphic to  $\mathbb{Z}_2$  as  $w$  maps  $\mathbb{R}P^2$  onto the non-trivial element of  $W(\mathbb{Z}_2) \cong \mathbb{Z}_2$ .  $\square$

*Remark.* An even simpler version of the argument of [1] implies that as a generalized homology theory

$$\mathcal{N}_n^{K\text{-Witt}}(X) \cong \bigoplus_{r+s=n} H_r(X; \mathcal{N}_s^{K\text{-Witt}})$$

for  $\text{char}(K) = 2$ , as in this case one no longer needs a separate argument to handle the odd torsion that can arise in  $H_n(X; \Omega_0^{K\text{-Witt}})$  as a result of  $\Omega_0^{K\text{-Witt}} \cong \mathbb{Z}$  not being 2-primary.

**Further discussion of oriented bordism.** We next provide some results that demonstrate the difficulty of determining which case of item (4) of Theorem 1 holds.

We will first see that  $w([M]) = 0$  for any  $\mathbb{Z}$ -oriented manifold: Since dimension mod 2 is the only invariant<sup>8</sup> of  $W(\mathbb{Z}_2)$ , this is a consequence of the following lemma, recalling that for a manifold,  $I^{\bar{m}}H_*(M) = H_*(M)$ .

**Lemma.** Let  $M$  be a closed connected  $\mathbb{Z}$ -oriented manifold of dimension  $4k + 2$ . Then  $\dim(H_{2k+1}(M; \mathbb{Z}_2)) \equiv 0 \pmod{2}$ .

*Proof.* By the universal coefficient theorem,

$$H_{2k+1}(M; \mathbb{Z}_2) \cong (H_{2k+1}(M) \otimes \mathbb{Z}_2) \oplus (H_{2k}(M) * \mathbb{Z}_2),$$

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<sup>8</sup>As observed in the proof of [7, Lemma III.3.3], rank mod 2 yields a homomorphism  $W(F) \rightarrow \mathbb{Z}_2$  for any field  $F$ . Since we know that  $W(\mathbb{Z}_2) \cong \mathbb{Z}_2$  and that  $\langle 1 \rangle$ , which has rank 1, is a generator of  $W(F)$  (it is certainly non-zero, using [7, Lemma I.6.3 and Lemma III.1.6]), it follows that rank mod 2 determines the isomorphism.

where the asterisk denotes the torsion product. Let  $T_*(M)$  denote the torsion subgroup of  $H_*(M)$ , and let  $T_*^2(M)$  denote  $T_*(M) \otimes \mathbb{Z}_2 \cong T_*(M) * \mathbb{Z}_2$ ; the isomorphism follows from basic homological algebra because  $T_*(M)$  is a finite abelian group.  $T_*^2(M)$  is a direct sum of  $\mathbb{Z}_2$  terms. Then  $H_{2k+1}(M) \otimes \mathbb{Z}_2 \cong \mathbb{Z}_2^B \oplus T_{2k+1}^2(M)$ , where  $B$  is the  $2k+1$  Betti number of  $M$ , and  $H_{2k}(M) * \mathbb{Z}_2 \cong T_{2k}^2(M)$ . Thus  $H_{2k+1}(M; \mathbb{Z}_2) \cong \mathbb{Z}_2^B \oplus T_{2k+1}^2(M) \oplus T_{2k}^2(M)$ . Since  $M$  is a closed  $\mathbb{Z}$ -oriented manifold, there is a nondegenerate skew-symmetric intersection form on  $H_{2k+1}(M; \mathbb{Q})$ , and so  $B$  is even. Since  $M$  is a closed  $\mathbb{Z}$ -oriented manifold, the nonsingular linking pairing  $T_{2k+1}(M) \otimes T_{2k}(M) \rightarrow \mathbb{Q}/\mathbb{Z}$  gives rise to an isomorphism  $T_{2k+1}(M) \cong \text{Hom}(T_{2k}(M), \mathbb{Q}/\mathbb{Z})$ , and since  $\text{Hom}(\mathbb{Z}_n, \mathbb{Q}/\mathbb{Z}) \cong \mathbb{Z}_n$ , it follows that  $T_{2k+1}(M) \cong T_{2k}(M)$ . Therefore  $T_{2k+1}^2(M) \cong T_{2k}^2(M)$ . Thus  $H_{2k+1}(M; \mathbb{Z}/2)$  consists of an even number of  $\mathbb{Z}_2$  terms.  $\square$

*Remark.* Since the lemma utilizes only integral Poincaré duality and the universal coefficient theorem, it follows that, in fact,  $w([X]) = 0$  for any  $IP$  space<sup>9</sup>; these are spaces that satisfy local conditions guaranteeing that intersection homology Poincaré duality holds over the integers and that a universal coefficient theorem holds (see [3, 10]).

A slightly more elaborate argument demonstrates that it is also not possible to have  $w([X]) \neq 0$  if  $X$  is a  $\mathbb{Z}$ -oriented  $\mathbb{Z}_2$ -Witt space with at worst isolated singularities:

**Proposition.** *Let  $X$  be a closed  $\mathbb{Z}$ -oriented  $4k+2$ -dimensional  $\mathbb{Z}_2$ -Witt space with at worst isolated singularities. Then  $w([X]) = 0$ .*

*Proof.* Since  $X$  has at worst point singularities, it follows from basic intersection homology calculations (see [2, Section 6.1]) that  $I^{\bar{m}}H_{2k+1}(X; \mathbb{Z}_2) \cong \text{im}(H_{2k+1}(M; \mathbb{Z}_2) \rightarrow H_{2k+1}(M, \partial M; \mathbb{Z}_2))$ , where  $M$  is the compact  $\mathbb{Z}$ -oriented PL  $\partial$ -manifold obtained by removing an open regular neighborhood of the singular set of  $X$ . We will show that if  $[z] \in \text{im}(H_{2k+1}(M; \mathbb{Z}_2) \rightarrow H_{2k+1}(M, \partial M; \mathbb{Z}_2))$ , then the intersection product  $[z] \cdot [z] = 0$ . It follows that the intersection pairing on  $I^{\bar{m}}H_{2k+1}(X; \mathbb{Z}_2)$  is split by [7, Lemma III.1.1], since then there can be no non-trivial anisotropic subspace. This implies that  $w([X]) = 0$  by the definition of the Witt group.

The following argument that  $[z] \cdot [z] = 0$  was suggested by “Martin O” on the web site MathOverflow [9]. By Poincaré duality, it suffices to show that  $\alpha \cup \alpha = 0$ , where  $\alpha$  is the Poincaré dual of  $[z]$  in  $H^{2k+1}(M, \partial M; \mathbb{Z}_2)$ . But now  $\alpha \cup \alpha = Sq^{2k+1}\alpha = Sq^1Sq^{2k}\alpha = \beta^*Sq^{2k}\alpha$ , where  $\beta^*$  is the Bockstein associated with the sequence  $0 \rightarrow \mathbb{Z}_2 \rightarrow \mathbb{Z}_4 \rightarrow \mathbb{Z}_2 \rightarrow 0$  (see [5, Section 4.L]). In the case at hand, this is the Bockstein  $\beta^* : H^{4k+1}(M, \partial M; \mathbb{Z}_2) \rightarrow H^{4k+2}(M, \partial M; \mathbb{Z}_2)$ . But this map is trivial. To see this, observe that there is a commutative

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<sup>9</sup>Also called “intersection homology Poincaré spaces,” though this is perhaps a misnomer as “Poincaré spaces” are generally not required to be manifolds while  $IP$  spaces are still expected to be pseudomanifolds.

diagram

$$\begin{array}{ccc}
H^{4k+1}(M, \partial M; \mathbb{Z}_2) & \xrightarrow{\beta^*} & H^{4k+2}(M, \partial M; \mathbb{Z}_2) \\
\downarrow \cong & & \downarrow \cong \\
H_1(M; \mathbb{Z}_2) & \xrightarrow{\beta_*} & H_0(M; \mathbb{Z}_2),
\end{array}$$

where  $\beta_*$  is the homology Bockstein and the vertical maps are Poincaré duality. The existence of this diagram follows as in [8, Lemma 69.2]. But now  $\beta_* : H_1(M; \mathbb{Z}_2) \rightarrow H_0(M; \mathbb{Z}_2)$  is trivial, as the standard map  $\times 2 : H_0(M; \mathbb{Z}_2) \rightarrow H_0(M; \mathbb{Z}_4)$  is injective.  $\square$

Hence any candidate to have  $w([X]) = 1$  must have singular set of dimension  $> 0$  and must not be an IP space. Given that all  $K$ -Witt spaces for  $\text{char}(K) \neq 2$  are  $K$ -Witt bordant to spaces with at worst isolated singularities [11, 1], it is unclear how to proceed to determine whether  $\mathbb{Z}_2$ -Witt spaces with  $w([X]) = 1$  exist. One method to prove that they do not would be to try to show “by hand” that every  $\mathbb{Z}_2$ -Witt space is  $\mathbb{Z}_2$ -Witt bordant to a space with at most isolated singularities, but the only proof currently known to the author of this fact for fields of other characteristics utilizes the bordism computations of [11, 1].

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